

Localized waves without the existence of extended waves: oscillatory convection of binary mixtures with strong Soret effect

D. Jung and M. Lücke
*Institut für Theoretische Physik,
 Universität des Saarlandes, Postfach 151150,
 D-66041 Saarbrücken, Germany*
 (Dated: February 8, 2008)

Spatially confined solutions of traveling convection rolls are determined numerically for binary mixtures like ethanol-water. The appropriate field equations are solved in a vertical cross-section of the rolls perpendicular to their axes subject to realistic horizontal boundary conditions. The localized convective states are stably and robustly sustained by strongly nonlinear mixing and complex flow-induced concentration redistribution. We elucidate how this enables their existence for strongly negative separation ratios at small subcritical heating rates below the saddle-node of extended traveling convection rolls where the quiescent fluid is strongly stable.

PACS numbers: 47.20.-k, 47.54.+r, 44.27.+g, 47.20.Ky

Many nonlinear dissipative systems that are driven sufficiently far away from thermal equilibrium show self-organization out of an unstructured state: A structured one can appear that is characterized by a spatially extended pattern which retains some of the symmetries of the system [1]. Some systems form also spatially confined structures [2]. In particular localized traveling wave (LTW) structures that are embedded at subcritical driving rates in the stable featureless surrounding and that compete with the subcritically bifurcated extended traveling wave (TW) pattern have been explored in experiments [3], numerical simulations of the appropriate field equations [4, 5], and via various model approaches [6, 7]. The coexistence of the TW state and the quiescent state is a prerequisite for interpreting LTWs as spatial connections of the former to the latter via two fronts. However, this picture coming from simple cubic-quintic amplitude equations is too simple for convection in binary mixtures: there the strongly nonlinear TWs cannot be described by a power-law expansion in one amplitude [8]. Here we show that LTWs are - independent of TWs - separate nonlinear states.

We predict and describe in quantitative detail stable subcritical LTWs in mixtures with strong Soret effect at small heating rates where no extended TW whatsoever – stable or unstable – exists [9]. Furthermore, we elucidate that the concentration field and its current are the key to understand how LTW convection is sustained when the quiescent conductive state is strongly stable. The latter is solutally stabilized since the Soret coupling between deviations δT of temperature and δC of concentration from their means increases (decreases) the lighter component in cold (warm) fluid regions thereby reducing the buoyancy. The most important effect is a flow-induced concentration redistribution within the LTW: It is the strongly nonlinear mixing by the LTW flow with large Peclet numbers that reduces the Soret-generated, stabilizing concentration gradients and that increases *locally* the driving buoyancy force to levels which suffice to cause convection in *well mixed* fluid with strongly reduced con-

centration variations [12].

We have solved the field equations [13] for convection in binary mixtures like ethanol-water for Lewis number $L=0.01$, Prandtl number $\sigma=10$, and separation ratios [1] $-0.25 \leq \psi \leq -0.4$ with a finite-difference method [14] in a vertical cross section through the convection rolls perpendicular to their axes. To simulate 1D patterns arising in narrow annular channels we applied laterally periodic boundary conditions at $x = 0, \Gamma$ with Γ up to 160 times the layer height to accommodate the largest LTWs. Spatially extended TW states of different wavelength $\lambda = 2\pi/k$ were obtained with $\Gamma = \lambda$. Horizontal boundaries at top and bottom, $z = \pm 1/2$, are realistic, i.e., no slip, perfectly heat conducting, and impermeable. Our control parameter $r = R/R_c^0$ measuring the thermal driving is the Rayleigh number R reduced by the critical one $R_c^0=1707.762$ for onset of convection in a pure fluid. To measure the lateral variation of the mixing in oscillatory convective flow we use the mixing number $M(x)$

$$M^2 = \overline{\langle (\delta C)^2 \rangle} / \overline{(\delta C_{cond})^2}. \quad (0.1)$$

Here overbars imply vertical averaging. Brackets denote temporal averaging, $\langle f \rangle = \langle f(x + v_d t, z, t) \rangle$, over one oscillation period in the frame comoving with the slow drift velocity v_d of the LTW envelope. In a perfectly mixed fluid M vanishes while $M \rightarrow 1$ in the conductive state [denoted by the subscript *cond* in Eq.(0.1)] with its large Soret-induced vertical concentration gradient.

In Fig. 1 we show how an increasing Soret coupling strength changes the bifurcation properties of LTW width l , maximal vertical flow velocity w_{max} , frequency ω , and LTW drift velocity v_d . The driving interval (r_{min}, r_{max}) with stable stationary LTWs moves upwards and its extension increases with $|\psi|$. At its lower end LTWs are for all ψ narrow pulses of universal structure and l of about 5. At the upper end l seems to diverge. LTWs with, say, $l \lesssim 7$ behave differently than the broad-width LTWs (cf. Fig. 2) with an extended TW like center part between the two limiting fronts. The change in the

LTW bifurcation diagrams of ω and v_d versus r in Fig. 1 reflects this difference. Our simulations indicate that the LTWs of Fig. 1 are uniquely selected, monostable confined solutions [15] and robust against small perturbations.

With increasing $|\psi|$ TW saddle-node location r_s^{TW} and oscillatory threshold r_{osc} move much faster to larger r than the lower LTW band edge r_{min} . At $\psi=-0.08$ (not shown here) one has $r_s^{TW} < r_{osc} < r_{min}$ so that there the narrow LTW pulses that have been investigated extensively lie in the linearly convectively unstable parameter region (see, e.g., [13] for discussion and references). However, already at $\psi=-0.4$ almost all LTWs appear *below* the TW saddle-node, i.e., ahead of the TW nose of Fig. 3. Thus, stable LTW convection is driven here at heating rates r for which any extended TW convection is impossible. Even at $\psi=-0.25$ the relevant LTW parameter combinations of $\omega, r, k_{plateau}$ (big bullets in Fig. 3) lie outside and ahead of the TW bifurcation surface. For all ψ investigated here the LTW width seems to diverge with increasing r when its frequency and plateau wave number $k_{plateau}$ (big bullets in Fig. 3) approaches an $\omega - r - k$ combination of a TW.

LTWs are sustained by the following complex, large-scale concentration redistribution process. While traveling from tail to head LTW rolls increase their lateral concentration contrast, i.e., M and with it v_p , and λ [16] (cf. Fig. 2). Positive "blue" (negative "red") concentration deviation from the global mean is sucked from the top (bottom) boundary layer into right (left) turning rolls as soon as they become nonlinear under the trailing front. This happens when the vertical velocity w roughly exceeds v_p [left arrow in Fig. 2(b)] so that regions with closed streamlines appear [8, 13]. Within them "blue" ("red") concentration is transported predominantly in the upper (lower) part of the layer to the right. Mean concentration, on the other hand, migrates mostly to the left along open streamlines that meander between the closed roll regions and that follow the border line between green and yellow in Fig. 2(a). The *time averaged* current of δC [green lines in Fig. 2 (e)] reflects the mean properties of this transport. Since "blue" and "red" ("yellow") is transported away from (towards) the left trailing front mean concentration accumulates there and causes a strong drop of $M(x)$. By the same token the leading front's concentration variations and with it

$M(x)$ are strongly increased even beyond the conductive state's values. Thus, unlike TWs LTWs do not reach a balance between δC injection and advective mixing and diffusive homogenization on a constant level of small M . Rather LTW rolls collapse under the leading front when v_p has grown up to w [right arrow in Fig. 2(b)]. Thereafter concentration is discharged and sustains a barrier of $\langle \delta C \rangle$ ahead of the leading front. When the mixing increases with w_{max} and r so does l since the slower growth of δC and M to the higher levels necessary for the leading front's transition to conduction requires longer and longer propagation lengths for the rolls.

The lateral redistribution causes the mean convectively generated C -profile [green line in Fig. 2(d)] to extend significantly further into the conductive region than the mean convective temperature field (red line). Thus, the buoyancy $\langle b \rangle$ [black line in Fig. 2(d)] is determined in the front regions predominantly by the concentration field. This explains (i) the decrease of buoyancy below conduction levels ahead of the leading front with the associated restabilization of conduction there and (ii) the increase of $\langle b \rangle$ out of the conductive state under the trailing front and its strong overshoot [cf. Fig. 2(d)] over the bulk enabling convection growth.

That the driving motor for LTW convection is located under the trailing front where $\langle b \rangle, M, v_p, \lambda$ are extremal can also be seen from the relaxation behavior after changing r : Not only $\langle b \rangle, M$, and v_p but also ω and v_d appropriate for the new r are almost instantaneously realized locally under the trailing front while the relaxation of the leading front involving also diffusive processes takes much longer time, cf., Fig. 4. Thus there ω and v_d is largely selected at the trailing front while the large-scale lateral concentration transport and redistribution determines the leading front's properties and its location, i.e., the LTW width.

To summarize: We have found, analyzed, and explained uniquely selected stable subcritical LTWs in mixtures with strong Soret effect at small heating rates where extended TWs are not possible, i.e., below the saddle node bifurcation of the latter. These strongly nonlinear LTWs are self-consistently sustained by a concentration redistribution such that flow-induced mixing increases locally the buoyancy to levels that suffice to drive well mixed fluid flow.

-
- [1] M. C. Cross and P. C. Hohenberg, Rev. Mod. Phys. **65**, 851 (1993) and references cited therein.
 - [2] J. Wu, R. Keolian, and I. Rudnick, Phys. Rev. Lett. **52**, 1421 (1984); A. J. Simon, J. Bechhoefer, and A. Libchaber, *ibid.* **61**, 2574 (1988); H. H. Rotermund, S. Jakubith, A. von Oertzen, and G. Ertl, *ibid.* **66**, 3083 (1991); W. S. Edwards and S. Fauve, J. Fluid Mech. **278**, 123 (1994); P. B. Umbanhowar, F. Melo, and H. L. Swinney, Nature **382**, 793 (1996); O. Lioubashevski, H. Arbell, and

- J. Fineberg, Phys. Rev. Lett. **76**, 3959 (1996); A. Groisman and V. Steinberg, *ibid.* **78**, 1460 (1997).
- [3] E. Moses, J. Fineberg, and V. Steinberg, Phys. Rev. **A35**, 2757 (1987); R. Heinrichs, G. Ahlers, and D. S. Cannell, *ibid.* 2761 (1987); P. Kolodner, D. Bensimon, and C. M. Surko, Phys. Rev. Lett. **60**, 1723 (1988); J. J. Niemela, G. Ahlers, and D. S. Cannell, *ibid.* **64**, 1365 (1990).
- [4] W. Barten, M. Lücke, and M. Kamps, Phys. Rev. Lett

- 66**, 2621 (1991); W. Barten, M. Lücke, M. Kamps, and R. Schmitz, Phys. Rev. E **51**, 5662 (1995).
- [5] H. Yahata, Prog. Theor. Phys. **85**, 933 (1991); L. Z. Ning, Y. Harada, and H. Yahata, *ibid.* **96**, 669 (1996).
- [6] M. Bestehorn, R. Friedrich, and H. Haken, Phys. Rev. A **42**, 7195 (1990); O. Thual and S. Fauve, J. Phys. (France) **49**, 1829 (1988); S. Fauve and O. Thual, Phys. Rev. Lett. **64**, 282 (1990); B. A. Malomed and A. A. Nepomnyashchy, Phys. Rev. A **42**, 6009 (1990); W. van Saarloos and P. C. Hohenberg, Physica D **56**, 303 (1992); R. J. Deissler and H. R. Brand, Phys. Lett. A **146**, 252 (1990); H. Riecke and W.-J. Rappel, Phys. Rev. Lett. **75**, 4035 (1995).
- [7] H. Riecke, Phys. Rev. Lett. **68**, 301 (1992); Physica D **61**, 253 (1992).
- [8] St. Hollinger, P. Büchel, and M. Lücke, Phys. Rev. Lett. **78**, 235 (1997).
- [9] Recently 2D subcritical localized states (so-called "worms") that could be generated from thermally driven fluctuations perturbing the conductive state have been seen in an electroconvecting nematic liquid crystal while experiment and theory indicate that extended convection bifurcates supercritically [10]. A weakly nonlinear model showed also subcritical worm-like solutions in the presence of a conjectured additional field that is advected obliquely from both sides towards the worms [11].
- [10] M. Dennin, G. Ahlers, and D. Cannell, Phys. Rev. Lett. **77**, 2475 (1996); U. Bisang and G. Ahlers, Phys. Rev. E **60**, 3910 (1999).
- [11] H. Riecke and G. D. Granzow, Phys. Rev. Lett. **81**, 333 (1998); A. Roxin and H. Riecke, Physica D **156**, 19 (2001).
- [12] The enhancement of the driving in the weakly nonlinear model of Riecke [7] is produced by an advected *mean* large-scale field that influences the growth of the critical mode(s). This model allows subcritical localized solutions – also for supercritical bifurcations to extended waves [7, 11].
- [13] M. Lücke, W. Barten, P. Büchel, C. Fütterer, St. Hollinger, and Ch. Jung, in *Evolution of Structures in Dissipative Continuous Systems*, edited by F. H. Busse and S. C. Müller, Lecture Notes in Physics, **m55** (Springer, Berlin, 1998), p. 127.
- [14] W. Barten, M. Lücke, M. Kamps, and R. Schmitz, Phys. Rev. E **51**, 5636 (1995).
- [15] The larger LTWs of the bistable ones found in relative short systems ($\Gamma = 20$ and 40) [4] are presumably stabilized by head-tail interactions.
- [16] M and $v_p = \omega\lambda/2\pi$ of stationary [8] and transient [17] TW solutions are practically proportional to each other. This roughly holds also locally in LTWs where M, v_p, λ vary spatially while ω is constant in the comoving frame.
- [17] C. Fütterer and M. Lücke, Phys. Rev. E **65**, 036315 (2002).
- [18] G. W. Baxter, K. D. Eaton, and C. M. Surko, Phys. Rev. A **46**, R1735 (1992); P. Kolodner, *ibid.* R1739 (1992); *ibid.* 6431 (1992); P. Büchel, Ph. D. thesis, Universität des Saarlandes, Saarbrücken, 1999.

FIG. 1: Bifurcation properties of LTWs (symbols) and TWs (lines) for different separation ratios ψ : (a) Full width l of LTWs at half maximum of the envelope of the vertical velocity field w [blue line in Fig. 2(b)]. For small filled symbols l kept on growing slowly. (b) Maximal vertical flow velocity w_{max} . (c) Frequency; for LTWs in the frame comoving with v_d . (d) Drift velocity v_d of LTWs. Phase velocity is always to the right, positive, and much larger. Thick lines in (b, c) denote TWs with saddle-node wave number $k_s^{TW} \simeq \pi$. Thin lines are TWs with plateau wave number $k_{plateau}$ of the last LTW before the $l \rightarrow \infty$ transition from LTW to TW. The frequency of the latter is determined in the frame comoving with the last LTW. Unstable TWs (dashed lines; determined with a control method) bifurcate subcritically with large Hopf frequency $\omega_H(k)$ at $r_{osc}(k)$ out of the conductive state and become stable (solid lines) at the saddle-node $r_s^{TW}(k)$ when lateral periodicity is imposed with $\Gamma = \lambda = 2\pi/k$.

FIG. 2: Broad LTW of $l=17.4$: (a) Snapshot of concentration deviation δC from global mean (light green/yellow) in a vertical cross section of the layer. (b) Snapshots of lateral wave profiles at midheight, $z=0$, of δC (green), vertical velocity w (blue), and its envelope. At the arrows $w_{max} = v_p$. (b) Mixing number M (green), Eq. (0.1) and phase velocity v_p (black) of nodes of $w(z=0)$ in frame comoving with v_d . The variation of $\lambda(x) = 2\pi v_p(x)/\omega$ is the same since the LTW frequency ω is a *global* constant. Thin (thick) bullet marks smallest (plateau) wavelength for discussion. (d) Time averaged deviations from the conductive state at $z=-0.25$ for concentration (green), temperature (red), and their sum ($\langle b \rangle$) measuring the convective contribution to the buoyancy. (e) Streamlines of time averaged concentration current $\langle \mathbf{J} \rangle = \langle \mathbf{u} \delta C - L \nabla (\delta C - \psi \delta T) \rangle$ (green) and velocity field $\langle \mathbf{u} \rangle$ (blue). The latter results from (b) and documents roll shaped contributions of $\langle \mathbf{u} \rangle \langle \delta C \rangle$ to $\langle \mathbf{J} \rangle$ under the fronts and the associated $\langle \delta C \rangle$ redistribution. Thick blue and green arrows indicate $\langle \mathbf{u} \rangle$ and transport of positive δC (alcohol surplus), respectively. Thus, in the lower half of the layer negative δC (water surplus) is transported to the right. Parameters are $\psi=-0.35$, $r=1.346$.

FIG. 3: TW and LTW oscillation frequencies over the $k - r$ plane. Extended TWs (nose shaped surface) start as unstable solutions with $\omega_H(k)$ at upper thick line – its projection onto the $k - r$ plane is the threshold curve $r_{osc}(k)$. Dent in upper nose part is related to appearance of closed w -streamlines when $w_{max} \simeq v_p$ causing δC -anharmonicities and the breakdown of small-amplitude expansions [8, 13]. With decreasing ω TWs become stable in periodic systems of $\Gamma = 2\pi/k$ at the saddle-node line (dotted outermost perimeter of the nose) located at $r_s^{TW}(k)$. For large Γ TWs become stable at the slightly larger Eckhaus boundary $r_E^{TW}(k)$ [18]. TWs with $k_s^{TW} \simeq \pi$ of the tip of the nose (plateau wave number $k_{plateau}$ of the last LTW before its $l \rightarrow \infty$ transition) are marked – as in Fig. 1 – by thick (thin) curves on the nose surface. Thin dotted horizontal lines denote LTWs: they contain for fixed ω a broad band of wave numbers extending from the largest ones, $k_{max} \simeq 4.3$ (small bullets as in Fig. 2), to small ones outside the plot range. Plateau wave numbers $k_{plateau}$ are marked as in Fig. 2 by big bullets that appear only where l is sufficiently large.

FIG. 4: Thick full (dashed) lines show front relaxation over a limited time interval after changing the driving from $r=1.41$ to 1.43 (1.38). Thin gray (black) lines are world lines of w -nodes (front positions) of the original LTW ($r=1.41$, $\psi=-0.4$, $l=20$).